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Review of Particle Packing Theories Used For Concrete Mix Proportioning

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Abstract – High performance concrete (HPC) has became more popular in recent years. The various performance attributes of HPC such as strength, workability, dimensional stability and durability against adverse environmental conditions, can be achieved by rationally proportioning the ingredients. Various methods have been in use for proportioning HPC mixes. Particle packing theories proposed by various researchers is an advanced step in this direction. This paper presents a review of these theories.

Index Terms – Angularity, Coarse Aggregate, Concrete Mix Proportioning, Digital Image Processing, Particle packing Theories, Shape, Surface Texture.

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1. INTRODUCTION

CONCRETE is the widely used construction material. It is produced by proper proportioning of ingredients such as cement, water, coarse aggregate and fine aggregate, so as to satisfy the required characteristics in green and hardened state. HPC have same constituents as that of concrete along with one of the following product such as organic admixture, supplementary cementitious materials, fibers etc and which are not limited to the final compressive strength, but include rheological properties, early-age characteristics, deformability properties and durability aspects. Thus the purpose of mix proportioning is to obtain concrete that will have suitable workability, maximum density, strength at specified age, dimensional stability and specified durability. Proportioning of concrete mixes is highly trial intensive. A purely experimental and empirical optimization could not give optimum proportion as number of parameters are involved as input and output as mentioned above. But the positive aspect is no concrete technology is younger technology. Huge amount of experimental data and various mix proportioning methods are available for designing the concrete. Concrete proportioning is first of all the packing problem. All existing methods recognize this problem by suggesting the measurement of the packing parameter of some component or by approximating an 'ideal' grading curves.

1.1 CONCRETE MIX PROPORTIONING AS PER VARIOUS CODES

Evolutions in concrete mix proportioning procedures are taking place since long. Abram's w/c ratio versus strength law is a breakthrough step (1918) [1]. Angularity number as suggested by Shergold [1] is a pioneering work in the evaluation of aggregate shape using the concept of percentage of voids. It is determined by subtracting the voids in the most rounded gravel from the voids present in the aggregate, when compacted in a specified manner. It is observed that the voids in the most rounded gravel are about 33%. The fact that mixture proportioning has long been more 'an art than a science' (Neville, 1995) [1] is illustrated by the variety of methods encountered worldwide.

Developments in methods of proportioning of concrete mixes.

1. Dreux 1970, this method is basically of an empirical nature, which was based upon Caquot's optimum grading theory.
2. DOE 1988 (Department of Environment, UK) method, the method of DOE revised in 1988 has considered water cement ratio with regard to compressive strength is clearly the most advanced investigation, but not all crushed aggregate gives the same contribution to compressive strength.
3. ACI Committee – 211.1.91 method, this method is probably one of the most popular worldwide. It is best mainly on the works of American researches (Abrams and Powers). The relationship between water/cement ratio and compressive strength is assumed to be unique. Hence if the diversities of aggregate nature and cement strength are cumulated, the compressive strength obtained for a given water/cement ratio may range from 1 to 2, in relative terms. Therefore, the prediction of water/cement ratio appears very crude.
4. IS 10262 – 1982, IS 10262 – 2009, many of the criteria of the method is just like ACI 211 [1].

Various concrete mix proportioning method make the provision regarding grading and size of aggregate. The aggregates are broadly classified as angular / rounded, crushed / uncrushed and accordingly separate values of water content for desired workability are specified "[2],[3],[4]". However the shape and surface texture of aggregate have significant effect on the property of the concrete produced, because it is the result of parameters, like type of parent rock, the forces to which it is subjected during and after its formation, and design and operation of crushing equipment. Hence, there is a need for proper quantification of aggregate for concrete mix proportioning. One major effect is on the packing density of aggregate which determine the amount of cement paste needed to fill the voids between the aggregate particles. Methods have been proposed that deal with the minimization of voids or the maximization of the packing density of aggregates or the dry components of mixtures. This paper presents a review of these theories.

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2. FUNDAMENTALS OF PARTICLE PACKING THEORIES

The packing of an aggregate for concrete is the degree of how good the solid particles of the aggregate measured in terms of ‘packing density’, which is defined as the ratio of the solid volume of the aggregate particles to the bulk volume occupied by the aggregate, as given by:

$$Packing\ Density\ (\phi) = \frac{Solid\ volume}{Total\ volume}$$

$$\phi = \frac{V_s}{V_t} = \frac{V_s}{V_s + V_v} = 1 - e \qquad (1)$$

Where :
Vs = volume of solids
Vt = total volume = volume of solids plus volume of voids
e = Voids = volume of voids over total volume to

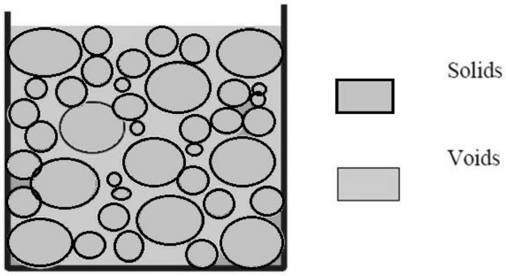

Fig. 1 Definition of Packing Density

From the packing density ‘voids ratio’, that is the ratio of the volume of voids between the aggregate particles to the bulk volume occupied by the aggregate.

Particle packing models are based on the concept that voids between larger particles would be filled by smaller particles thereby reducing the volume of voids or increasing the packing density. Thus the important property regarding packing of multi particle system is the packing density as per figure 1.

The packing density of a multiparticle system is of basic importance in science and industry. Efficient packing in the making of ceramics has undoubtedly interested mankind for centuries. More recently, a greater knowledge of packing would prove useful to the concrete and nuclear power industries as well as in physics and soil mechanics.

The particle packing models may be categorized as (a) discrete model (b) continuous model.

2.1 DISCRETE MODEL

The fundamental assumption of the discrete approach is that each class of particle will pack to its maximum density in the volume available [5]. The discrete model is classified as (i) binary (ii) Ternary and (iii) Multimodal mixture model.

2.2 BINARY MIXTURE MODEL

Basic research of packing theory was started by Furnas [6]. His theory was set up for sphere shaped particles and was based on the assumption that the small particles fill out the cavities between the big particles without disturbing the packing of the big particles. Furnas considered the ideal packing of a mixture of two materials. Depending upon the volume fraction of fine and coarse aggregate, two cases may be considered

i. The volume fraction of small particle is large ($y_1 >> y_2$). This case is called “fine grain dominant”.
ii. The volume fraction of coarse particle is large ($y_2 >> y_1$). This case is called “coarse grain dominant”.

This two cases is only possible when $d_1 << d_2$ (d_1 and d_2 being the particle diameters). If this condition is not fulfilled, the packing density of the binary mixtures will also depend on the diameter ratio d_1 / d_2. When the diameter $d_1 \approx d_2$ the interaction effect occurs. The effect is classified as wall effect and loosening effect.

Wall effect: - when an isolated coarse particle is in the matrix of fine aggregates it disturbs the packing density of fine aggregate. There increased voids around the fine particles causing wall effect.

Loosening effect: - when a fine particle is in the matrix of coarse particle and the small particle is too large to fit into the interstices of the coarse aggregate ($d_1 \approx d_2$) it disturbs the packing density of coarse particles.

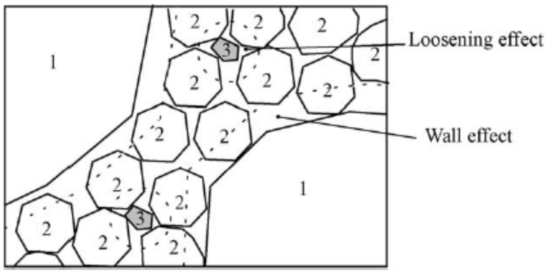

Fig. 2 Wall Effect and Loosening Effect

M. Mooney [7] Einstein's viscosity equation for an infinitely dilute suspension of spheres is extended is apply to a suspension of finite concentration. The argument makes use of a functional equation which must be satisfied, if the final viscosity is independent of stepwise sequence additions of partial volume fractions of the spheres to the suspension. For a monodisperse system the solution of the functional equation is $\eta_r = \exp\left(\frac{2.5\phi}{1-k\phi}\right)$ where η_r is the relative viscosity, ϕ the volume fraction of the suspended spheres, and *k* is a constant, the self-crowding factor, predicted only approximately by the theory. The solution for a polydisperse system involves a variable factor, λ_{ij}, which measures the crowding of spheres of radius r_j by spheres of radius r_i. The variation of λ_{ij} with r_i/r_j is roughly indicated. There is good agreement of the theory with published experimental data.

T. C. Powers [8] in his studies on particle packing took account of the wall effect and loosening effect. He proposed an expression to get the minimum void ratio of the binary mixture.

Aim and Goff “[9],[10]” proposed a simple geometrical model to account for the excess porosity observed experimentally in the first layer of spherical grains in contact with a plane and smooth wall, the work of Aim and Goff addressed the “wall effect” and suggested a correction factor when calculating the packing density of binary mixtures.

2.3 TERNARY MIXTURE MODELS

Toufar *et al* “[9], [10]” extended the binary mixture model to calculate the packing density. The fundamental concept of the Toufar model is that the smaller particles (diameter ratios > 0.22) will actually be too large to be situated within the interstices between the larger particles. The result is a packing of the matrix that may be considered as (i) a mixture of packed areas mainly consisting of larger particles and (ii) packed areas that may mainly consisting of larger particles and (ii) packed areas that may mainly consist of smaller particles with larger particles distributed discretely throughout the matrix of smaller particles.

For a multi-component system, it is assumed that any two components form binary mixtures. Then the packing density for the total multi-component mixture is calculated by summation of the contribution from all the binary mixtures.

Goltermann *et al* "[10],[11]" proposed a modification in Toufar model. They also termed the packing degree factor of the individual components (ϕ_1 and ϕ_2) as "Eigen Packing", which is calculated according to the procedure mentioned in their work.

Goltermann et al also compared the packing values suggested by Aim model, Toufar model and Modified Toufar model to the experimental packing degree of the binary mixtures. They found that Toufar model, especially the modified Toufar model, corresponded very well to the measured packing degrees. Europack is software based on modified Toufar model. For this model, the required input information is density, packing density, and characteristic diameters of each components. The characteristic diameter is defined as the diameter for which the cumulative probability of the Rosin-Raimmler distribution is 0.37. This corresponds approximately to the size associated with 63 percent of the material passing. With the size distribution, the model determines the characteristic diameter.

2.4 MULTI-COMPONENT MIXTURE MODELS

Based on the property, of multimodal discretely sized particles, De Larrad postulate different approaches to design concrete; the Linear Packing Density Model (LPDM), Solid Suspension Model (SSM) and Compressive Packing Model (CPM) "[2],[13]".

T. Stovall *et al* [13] this paper presents a model for the packing density of multi sized grains. For a given mixture, the packing density is expressed as a function of the fractional solid volume of each grain size present. The case of grain sizes continually distributed is derived. Comparison of model predictions with binary, ternary and higher-order mixtures is quite encouraging. They claimed that LPDM showed good performances in predicting optimal proportions of superplasticised cementitious materials

F. de Larrard *et al* "[14],[15],[16],[17],[18],[19]" the concept of high packing density has been recently rediscovered, as a key for obtaining ultra-high-performance cementitious materials. These model are derived from the Mooney's suspension viscosity model. De Larrard and Sedran proposed the solid suspension model (SSM) with some modification in LPDM. They concluded that SSM is a valuable tool to optimize high packing density of cementitious materials. The essential innovation is the distinction between the actual packing density, ϕ, and virtual packing density, β - the maximum packing density achievable with a given mixture, by keeping each particle in its original shape and placed one by one of a mixture. It was also anticipated that the model would be suitable for predicting the plastic viscosity of concentrated suspensions.

M. Glavind, et al [16] When selecting a concrete mix design, it is always desirable to compose the aggregates as densely as possible, i.e. with maximum packing. That minimises the necessary amount of binder which has to fill the cavities between the aggregates for a constant concrete workability. Apart from an obvious economic benefit, a minimum of binder in concrete results in less shrinkage and creep and a more dense and therefore probably a more durable and strong concrete type.Another extended application of LPDM has been by Glavind et al. They used the concept of "Eigen packing" to calculate the packing density.

De Larrard "[2],[13]" presumed that the packing density of the mixtures depends also on the process of the building of the packing, such as compaction effort, the proposed the compressible packing model (CPM). This model was derived from the linear packing model proposed by Lee and is independent of other models (that is, LPDM and SSM). He introduced the index K, to calculate actual packing density, ϕ from virtual packing density, β.

J. D. Dewar "[20],[21]" consider packing density in loose condition. The parameter requires for this model is the mean size (i.e. grading) and the density of each fraction. Dewar suggest that mean diameter of micro fines and cementitious material could be estimated from the Blaine fineness not if the size distribution is not available. Theory of particle mixture (TPM) works with void ratio instead of packing density, where void ratio as defined as the ratio of voids to solids volume. The relationship between voids ratio, U and packing density
ϕ is

$$u = \frac{1}{\phi} - 1 \qquad (2)$$

2.5 CONTINUOUS MODELS

Continuous approach assumes that all possible sizes are present in the particle distribution system, that is, discrete approach having adjacent size classes ratios that approach 1:1 and no gaps exist between size classes.

The fundamental work of Féret *et al* [5], Fuller *et al* [12] showed that the packing of concrete aggregates is affecting the properties of the produced concrete. Both Féret as well as Fuller and Thomsen concluded that the continuous grading of the composed concrete mixture can help to improve the concrete properties. Féret demonstrated that the maximum strength is attained when the porosity of the granular structure is minimal. In 1907 Fuller and Thomson proposed the gradation curves for maximum density, which is well known as Fuller's "ideal" curve. It is described by a simple equation:

$$CPFT = (d / D)^n 100 \qquad (3)$$

Where,
CPFT = cumulative (volume) percent finer than,
n = 0.5; the value of n was later revised to 0.45; these curves find application in highway pavement mixture design.

The above expression was recently modified by Shakhmenko and Birsh for concrete mixture proportioning as follows:

$$CPFT = T_n (d_i / d_0)_n \qquad (4)$$

Where,
n = degree of an "ideal" curve equation
T_n = is a coefficient, dependent on maximum size of aggregate and the exponent n.

Andreassen *et al* "[9], [10]" worked on the size distribution for particle packing with a continous approach and proposed the "Andressen equation" for ideal packing. Although the approach is more theoretical, it partly represents an empirical theory of particle packing.

Andressen assumed that the smallest particles would be infinitesimally small. Dinger and Funk recognized that the finest particles in real materials are finite in size and modified the Andreassen equation considering the minimum particle size in the distribution. A modified model linking the Andreassen and Furnas distributions was later developed and termed as AFDZ (Andreassen, Funk, Dinger and Zheng) equation for dense packing.

According to the Andressen model,

$$CPFT = (d / D)^n 100 \qquad (5)$$

According to the Modified Andressen model,

$$CPFT = \{(d - d_0) / (D - d_0)\}^q 100 \qquad (6)$$

Where,
CPFT = the cumulative (volume) percent finer than,
d = the particle size,
d_0 = the minimum particle size of distribution,
D = the maximum particle size, and
q = the distribution coefficient or exponent.

The exponent, q, in the Andressen equation could be varied from 0.21 to 0.37, depending upon the various workability requirements. If the exponent increases, it means an increase of the coarse materials, and if it decreases, the amount of the fine materials is increased [10]. The exponent value, q, gives the indication of the finer fraction that could be accommodated in the mixture. As the water demand and water holing capacity of the mixture is controlled by the volume of fines, this exponent gives a reasonable basis for choosing the amount of water and rheology modifying agents like superplasticiser to be added to the mixture.

The exponent value q = 0.25 to 0.3 may be taken for high performance concrete and conventional concretes depending compacting concretes, $q < 0.23$ may be taken, and for roller compacted concrete, $q > 0.32$ may be taken.

Rosin-Rammler Model

The characteristic diameters of the particle size distributions for the components of concrete were shown to be adequately

Described by the D' from the Rosin-Rammler equation which is written as:

R (D) = the residue fraction (percentage passing)
D = diameter
D' = characteristic diameter
n = constant, ranging from 1.04 – 4, usually between 1 and 2.

Johansen *et al* [10] have used this equation for finding out the characteristic diameter of the distribution for calculating the packing density of the mixtures in their discrete approach.

2.6 3D COMPUTER SIMULATION MODEL

Simulation to assess the packing characteristics has been developed based on static simulation system by Bentz et al and some system based on dynamic simulation system such as SPACE (software package for the assessment of compositional evaluation) by Stroeven *et al* "[22],[23],[24]".The SPACE system has been developed to assess the characteristics of dense random packing situation in opaque materials by a realistic structural simulation.

Grading of aggregate based on size and shape has significant effect on the properties of concrete produced. But all packing models are based on the assumption that particle are spherical. Kwan *et al* "[25],[26],[27]" the shape factor and convexity ratio are the important shape parameter. Void ratio, specific gravity and mean size of particle are important parameters influencing the packing density of mixture. Digital image processing and Fourier analysis are used to explore the characteristics of aggregate.

2.7 DIGITAL IMAGE PROCESSING

Rajeswari *et al.* "[28],[29]" also stated that the improvement in the shape of crushed rocks used as aggregates as amongst the most important characteristics of high quality aggregates particularly for use in the concrete or construction industry. Aggregates with beefed up characteristics such as more cubical and equidimensional in shape with better surface texture and ideal grading are considerably gaining much more attention particularly from the concrete industry as these aggregates greatly assist in increasing the strength and enhancing the quality of concrete. This work also scientifically showed the optimum orientation and packing of high quality shape aggregate particles (i.e. cubical and angular) in a concrete mix compared to the poorly shaped particles (i.e. irregular, elongated, flaky and flaky and elongated). Hence, aggregates with improvement in particle shape and texture acts as a catalyst for the development of good mechanical bonding and interlocking between the surfaces of aggregate particles in a concrete mix. Overall, stronger aggregates with improvement in particle shape and textural characteristics tend to produce stronger concrete as the weak planes and structures are being reduced. Substitution of equidimensional particles derived as crushed product produce higher density and higher strength concrete than those which are flat or elongated because they have less surface area per unit volume and therefore pack tighter when consolidated.

A concrete mix is constituted largely of aggregate and its quality is hence dependent on the grading, size, and shape of the aggregate used. Applications of the DIP technique to particle size and shape analysis have been attempted by Barksdale et al., Li et al., Yue and Morin, and Kuo et al., A.K.H. Kwan[25] any useful results have been obtained. The shape of the aggregate particles used has significant effects on the properties of the concrete produced. One major effect is on the packing density of the aggregate which determines the amount of cement paste needed to fill the voids between the aggregate particles. In order to study how the various shape parameters of aggregate particles would affect the packing of aggregate, aggregate samples of different rock types from different sources have been analysed for their shape characteristics using a newly developed digital image processing technique and their packing densities measured in accordance with an existing method given in the British Standard. The packing densities of the aggregate samples are correlated to the shape parameters to evaluate the effects of the various shape parameters on packing. From the results of the correlation, it is found that the shape factor and the convexity ratio are the most important shape parameters affecting the packing of an aggregate. Two alternative formulas revealing the combined effects of these two shape parameters on the packing density of aggregate are proposed.

However, there are a number of problems associated with the application of DIP to particle size and shape analysis. Traditionally, standard techniques and test procedures complying with British Standards, American Society for Testing and Materials (ASTM) and New Zealand Standards have been widely used to analyze and evaluate the shape, size grading and surface texture of aggregates.

Digital video technology has advanced so rapidly that it is now much more affordable and easier to use than before. From a video camera, a scene can be captured electronically producing video signals, which are first digitized and then stored as an array of pixels. Subsequently, pictorial information about the scene may be extracted from the pixel array by the use of a technique called digital image processing (DIP).

Over the past 20 years, many works have been done to improve the methods for analyzing aggregate images using digital image processing (DIP) technique particularly to shorten the time for classification thus making it more cost effective and faster compared to the conventional processes. Much of the work tried to explore the advantages of DIP to have a real time classification system and the data information storage for the aggregates, making it more automated thereby simplifying the analysis in the future. Different methods and algorithms were developed to tackle the issues encountered and to improve the process further. Kwan *et al* [27] adopted DIP to analyze the shape of coarse aggregate particles. Application of DIP for the measurement of coarse aggregate size and shape is presented in the works of Maerz *et al* [29]. Mora and Kwan [27] had developed a method of measuring the sphericity, shape factor and convexity of coarse aggregate for concrete using DIP technique.

A number of methods using imaging systems and analytical procedures to measure aggregate dimensions are already available. An imaging system consisting of a mechanism for capturing images of aggregates and methods for analyzing aggregate characteristics have been developed such as Multiple Ratio Shape Analysis (MRA), VDG-40 Video grader by Emaco Ltd Canada, Computer Particle Analyzer (CPA) by Tyler, Micromeritics Opti Sizer (PSDA) by Strickland, Video Imaging System (VIS) by John B. Long Company, Buffalo Wire Works (PSSDA) by Penumadu, Camsizer by Jenoptik Laser Optik System and Research Technology, Wip Shape by Maerz and Zhu ,University of Illinois Aggregate Image Analyzer (UIAIA) by Tutumluer et al. Aggregate Imaging System (AIMS) by Masad and Laser-Based Aggregate Analysis System (LASS) by Kim et al. Description of the existing test methods can be found in Al-Rousan "[30]-[31]". X. Jia *et al* [32] developed packing algorithm based on digitization technology that is "DigiPac" for non spherical partials of uniforms size and powders of different size distribution. The porosity obtained is consistent with the measurement of other model predictions. X-ray tomography is used for digitization of irregular shapes so that 3D images of a real particles are easily obtained. Since interactions of particles are limited to geometric constraints the limitation of DigiPac are obvious. The potential application of DigiPac may be found in ceramics, powder storage, transportation etc. more work needs to be done to extended DigiPac to solve more complicated system such as particles where cohesive forces are involved.

The packing density of aggregate can be measured under dry condition, due to agglomeration, all early attempts to measure the packing density of cementitious materials under dry condition failed. To overcome the above difficulty, The University of Hong Kong has recently developed a wet packing test for measuring the packing density of cementitious materials under wet condition. Wong and Kwan "[33],[34],[35]" developed wet packing density. Basically, this test mixes the cementitious materials with different amounts of water and determines the highest solid concentration achieved as the packing density of the cementitious materials. Any air trapped inside the cement paste is taken into account in the calculation of the packing density. If there is SP added to the cement paste, the effect of SP is also taken into account by adding exactly the same dosage of SP into the mixture. The accuracy of the wet packing test has been verified by Fung *et al* [36] checking against established packing models and the results indicated that the differences between theoretical results by packing models and experimental results by the wet packing test are well within 3%.

2.8 DISCRETE ELEMENT MODELING (DEM)

Piets stroeven "[37],[39]", Shihui shen, Hunan yu [40] suggest discrete element modeling(DEM) simulation method for particle packing analysis Contact force chains and mean contact force were calculated using PFC3D DEM simulation, which provided an indication of the capability of the aggregate structure to transmit stresses through aggregate skeleton, and thereby, to resist permanent deformation. The study conducted here demonstrated the aggregate size distribution played a significant role in the packing characteristics, affecting both volumetric and the contact characteristics of a packed structure. Such findings are critical for evaluating the combined effect of size and shape distribution on packing, and achieving a performance based aggregate gradation design.

3 CONCLUSION

The review of the research work shows that all the popular packing models are based on the assumption that the particles are spherical. Actually review studies have shown that shape factor and convexity ratio are the most important shape parameters and mean size, specific gravity and voids ratio are the most important size parameters influencing the packing of aggregate. Packing of aggregate seems to be sound concept to predict the behavior of fresh concrete and hardened concrete. A concrete mix is constituted largely of aggregate and its quality is hence dependent on the grading, size, and shape of the aggregate used.

REFERENCES

[1] A.M. Neville, *Properties of Concrete*, 4th ed., Longman, UK, 1997.
[2] IS: 10262, *Recommended guidelines for concrete mix design, Bureau of Indian Standards, New Delhi, India*, 1982.
[3] ACI 211.1-91, *Standard Practice for selecting proportions for normal, heavy weight, and mass concrete, Detroit, Michigan, USA*, 1994.
[4] *Department of Environment, Design of normal concrete mixes, Department of Environment, Building Research Establishment, Watford, UK*, 1988, 42 pp.
[5] R. Féret, *Sur la compactié des mortiershyrdauliques, Ann. Ponts Chaussée, mé.moires etdocuments*, Série 7, no. IV, 1892, pp. 5–164, (in French).
[6] C. C. Furnas, *"Grading Aggregates I-Mathematical Relations for Beds of Broken Solids of Maximum Density"*, U. S. Bureau of Mines. Vol. 23, No. 9. April 1931, pp 1052-1058.
[7] M. Mooney, *"The Viscosity of a Concentrated Suspension of Spherical Particles"*, Presented at the Annum Meeting of the Society of Rheology, New York, N. Y., November, 3-4, 1950, pp 162-170.
[8] Powers, T.C., *The Properties of Fresh Concrete*, John Wiley and Sons, 1968, New York, USA.
[9] Andersen, P.J. and Johansen, V. *Particle packing and concrete properties*, Material Science of Concrete: II, 1991, Skalny J and Mindess S (Edited), The American Ceramic Society, Inc., Westerville, Ohio. Pp.111-147.
[10] Senthilkumar V, Manu Santhanam, "*Particle packing theories and their applicationinconcrete mixture proportioning: a review"* The Indian Concrete Journal 2003:1324–31.
[11] Golterman, P., Johansen, V., Palbfl, L., *"Packing of Aggregates: An Alternative Tool to Determine the Optimal Aggregate Mix,"* ACI Materials Journal, Vol. 94, No. 5, 1997, p 435.
[12] Fuller, W.B. and Thompson, S.E., *The Laws of Proportioning Concrete*, American Society of Civil Engineers, Vol.33, 1907, pp.223-298.

[13] T. Stovall, F. De Larrard and M. Buil, *“Linear Packing Density Model of Grain Mixtures”*, Powder Technology, 48 (1986), pp 1 – 12.

[14] Franqois De Larrard, *“Ultrafine Particles for the Making of Very High Strength Concretes”*, cement and concrete research. Vol. 19, 1989, pp. 161-172.

[15] F. De Larrard and T. Sedran, *“Optimization of Ultra-High-Performance Concrete by the use of a Packing Model”* Cement and Concrete Research, Vol. 24, No. 6, 1994, pp.997-1009.

[16] M. Glavind, E. J. Pedersen, *“Packing Calculations Applied for Concrete Mix Design”* Proceedings Creating with Concrete, May 1999, University of Dunde, pp.1-10.

[17] Francois De Larrard, Thierry Sedran, *“Mixture-proportioning of high-performance concrete”,* Cement and Concrete Research 32 (2002) pp.1699–1704.

[18] De Larrard, et al, *“A New Rheometer for Soft-to-Fluid Fresh Concrete,”* ACI Materials Journal, Vol. 94, No. 12, 1997, pp. 234.

[19] De Larrard, F., *“Why Rheology Matters?”* Concrete International, Vol. 21, No. 8, 1998, p. 79.

[20] Dewar, J. D., *Computer Modeling of Concrete Mixtures,* E and FN Spon, London, 1999.

[21] Dewar, J.D. and Anderson, R., *Manual of Ready-Mixed Concrete, Blackie Academic and Professional,* 2nd ed., 1992, London.

[22] Piet Stroeven, MartijnStroeven, *“Assessment of packing characteristics by computer simulation”,* Cement and Concrete Research 29 (1999) pp.1201–1206.

[23] Piet Stroeven, Huan He, ZhanqiGuo, Martijn Stroeven, *“Particle packing in a model concrete at different levels of the microstructure: Evidence of an intrinsic patchy nature”,* Materials Characterization 60 (2009) pp.1088–1092.

[24] P. Stroeven, J. Hu, Z. Guo, *“Shape assessment of particles in concrete technology: 2D image analysis and 3D stereological extrapolation”,* Cement and Concrete Composites 31 (2009) pp.84–91.

[25] Kwan, A. K. H., Mora, C. F. and Chan, H. C. “*Particle shape analysis of coarse aggregate using digital image processing*”, Cement and Concrete Research, 29(9), (1999), pp. 1403-1410.

[26] C.F. Mora, A.K.H. Kwan, *“Sphericity, shape factor, and convexity measurement of coarse aggregate for concrete using digital image processing”,* Cement and Concrete Research 30 (2000) pp.351-358.

[27] A.K.H. Kwan, W.W.S. Fung, *“Packing density measurement and modelling of fine aggregate and mortar”*, Cement and Concrete Composites 31 (2009) pp.349–357.

[28] Rajeswari, S. R. M. *“Studies on the Production and Properties of Shaped Aggregates from the Vertical Shaft Impact Crusher,”* Master Thesis, School of Mineral and Material Resources Engineering, 2004.

[29] Mohammad Subhi Mohammad Al-Batah, *“A Novel Aggregate Classification Technique Using moment Invariants and Cascaded Multilayered Perceptron Network”*, International Journal of Mineral Processing, (7 March 2009). pp. 1–34.

[30] Erol Tutumluer, Tongyan Pan, Samuel h. Carpenter, *“Investigation of Aggregate Shape Effects on Hot Mix Performance Using an image Analysis Approach”,* Final Report on the TPF-5 (023) Department of Civil and Environmental Engineering, University of Illinois At Urbana-Champaign Urbana, Illinois, (2005), pp.1-121.

[31] M. R. Jones, L. Zheng and M. D. Newlands, *“Comparison of Particle Packing Models for Proportioning Concrete Constituents for Minimum Voids Ratio”,* Materials and Structures/Mat6riaux et Constructions, Vol. 35, June 2001, pp.301-309.

[32] X. Jia, R.A. Williams, *“A Packing Algorithms for Particles of Arbitrary Shapes”,* Powder Technology 120 (2001) pp.175-186.

[33] Henry H.C. Wong and Albert K.H. Kwan, *“Packing Density: A Key Concept for Mix Design of High Performance Concrete”,* 2007, pp.1-15.

[34] Henry H. C. Wong. Albert K. H. Kwan, *“Packing density of cementitious materials: part 1-measurement using a wet packing method”,* Materials and Structures (2008) 41: pp.689–701.

[35] K. H. Kwan. H. H. C. Wong, *“Packing density of cementitious materials: part 2-packing and flow of OPC + PFA + CSF”,* Materials and Structures (2008) 41: pp.773–784.

[36] W.W.S. Fung, A.K.H. Kwan, H.H.C. Wong, *“Wet packing of crushed rock fine aggregate”,* Materials and Structures (2009) 42: pp.631–643.

[37] Jing Hu and Piet Stroeven, *“Properties of the Interfacial Transition Zone in Model Concrete”,* Interface Science 12, 2004, pp.389-397.

[38] M. Gan, N. Gopinathn, X. Jia and R.A. Williams, *“Predicting Packing Characteristics of Particles of Arbitrary Shapes”,* Kona No. 22, (2004), pp. 82-92.

[39] P. Stroeven, J. Hu, *“Gradient structures in cementitious materials”,* Cement and Concrete Composites 29 (2007) pp.313–323.

[40] ShihuiShen, Huanan Yu, *“Characterize packing of aggregate particles for paving materials: Particle size impact”,* Construction and Building Materials xxx (2010) xxx–xxx.